Grade Level/Course: Grade 3
Lesson/Unit Plan Name: Comparing Fractions
Rationale/Lesson Abstract: Students develop an understanding for the role of the denominator as an indicator of the size of the unit as they compare fractions with the same numerator or denominator. They begin to see the relationship of those fractions to one half.
Timeframe: 5 to 8 days
Common Core Standard(s): CCSS.Math.Content.3.NF.A.3d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. ([1] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.) CCSS.Math.Content.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.([1] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)

Instructional Resources/Materials: Paper, pencil, straight edge, fraction tiles

Activity/Lesson:

These lessons assume that students have drawn bar models and plotted fractions on number lines.

Parts 1 through 3 are for grade 3, but can be used in grade 4 as an introduction. Comparisons to 1 half are incorporated throughout.

Part 1: Like Denominators (Compare Same Size Units)

Part 2: Compare Unit Fractions with Unlike Denominators (Different Size Units)

Part 3: Compare Fractions with Unlike Denominators and Like Numerators (Same Number of Different Size Units)

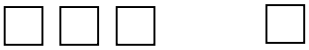
Next Steps: Part 4 is an overview of how grade 4 takes fractions to the next level: Compare Fractions with Unlike Denominators and Unlike Numerators.

Part 1 Overview: Like Denominators

Connection to Base Ten Concepts

5 tens < 7 tens

3 hundreds > 1 hundred

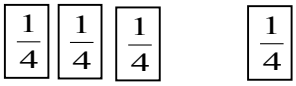


7 ones > 2 ones

Use What You Know: Compare Same Size Units

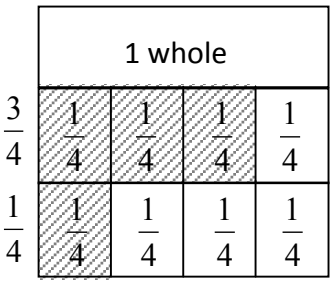
$\frac{3}{4}$ > $\frac{1}{4}$

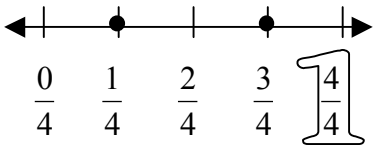
3 fourths > 1 fourth

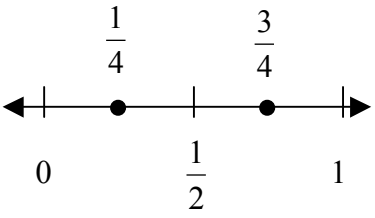


I know all the units are fourths, so they are all the same size. Three fourths is greater than 1 fourth.

Bar Model and Number Lines



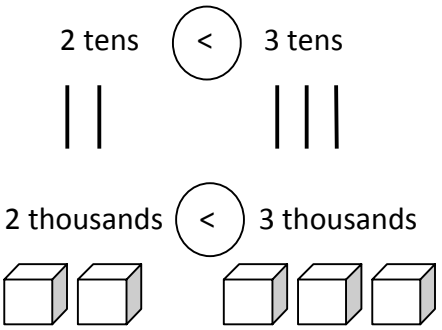




Part 1: Like Denominators

We've been working with fractions for a while. Now we are going to *compare* fractions.
You've already compared GREATER numbers like 100 and 1,000.
Let's talk about what you already know about comparing greater numbers.

You already know that 2 tens are less than 3 tens.
You already know that 2 thousands are less than 3 thousands.

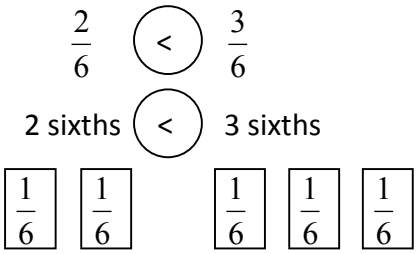


What's the same about the two sides? Share with your neighbor. I need a quiet hand to share.
(In the first, they are both tens. In the second, they are both thousands.)
What's different about the 2 sides? Share with your neighbor. Quiet hand to share.
(One side has only 2 objects. The other has 3)

In either of the pictures, are there tens on one side and thousands on the other? (no)
You are correct. That's because we have been comparing the same size units: tens to tens and thousands to thousands. That can make comparing easier, because all we have to do is count the number of units on each side.

Let's apply this idea to fractions.

If we had 2 sixths on one side and 3 sixths on the other, which would be greater? (3 sixths)
Of course! *Use tiles and/or drawing to show.*



So already we can compare some fractions without doing much work. If the denominator is the same for both fractions, we know that the units (or pieces) are the same size. Which part of the fraction is the denominator? (the bottom number)
Then all we have to do is compare the numerator, which tells us the number of units. Where is the numerator? (the number on the top)

Using Fraction Tiles (As Necessary)

We Do: Compare $\frac{3}{4}$ and $\frac{1}{4}$.

What is the denominator? (4) So we are working wit. fourths. We have 3 fourths and 1 fourth. Raise your hand if you know which is greater, (hands) Which one? (3 fourths) Good. Let's write down your thinking.

Possible Student Notes:

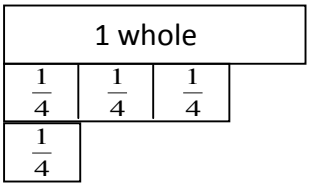
Compare: $\frac{3}{4}$ and $\frac{1}{4}$.

$\frac{3}{4}$ > $\frac{1}{4}$

3 fourths > 1 fourth

✓

Students prove answers with fraction tiles.



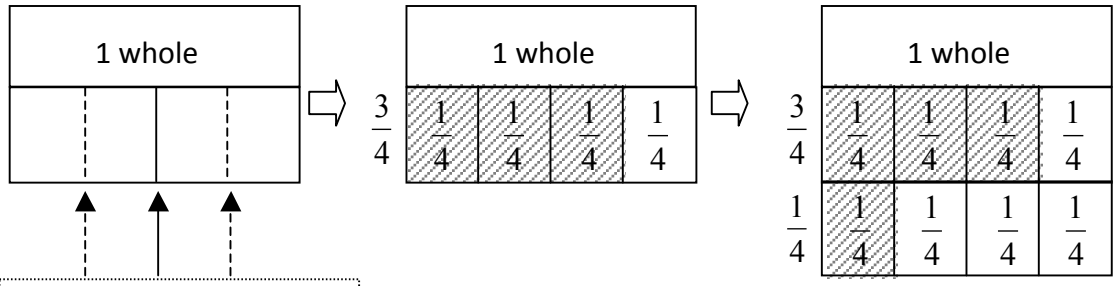
The check mark indicates that they have proven their answer with fraction tiles.

You Tries: *Have students work with partners or on their own to compare several pairs of fractions using the same process you just did with thems*

*3rd grade denominators are 2, 3, 4, 6, and 8. Fourth grade denominators are 2, 3, 4, 5, 6, 8, 10, 12, and 100. You can include 4th grade denominators in your you tries as an extension.

Model Drawings and Number Lines

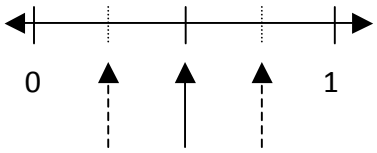
We Do: $\frac{3}{4} > \frac{1}{4}$ **(from the day before). This time make model drawings and number lines.**



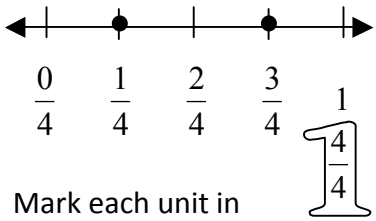
Let's start by dividing the whole into 2 equal pieces. Now what could we do? If I divide each piece into 2, would I have 4 pieces altogether?

Included in the lesson are bar model outlines to get started if needed, but students should also be drawing their own models. Student drawings do not have to be perfect. This is a chance to develop spatial awareness and fine motor skills.

Number Lines:



Establish 0, 1, and halfway mark. Then divide each half into 2 equal pieces.

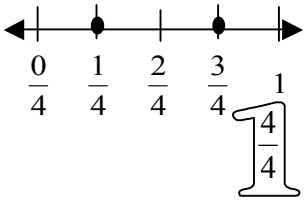


Mark each unit in fourths. Plot 1 fourth and 3 fourths.

Student Notes:

$\frac{3}{4} > \frac{1}{4}$ or $\frac{1}{4} < \frac{3}{4}$

	1 whole			
$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$



You Tries for Model Drawings and Number Lines:

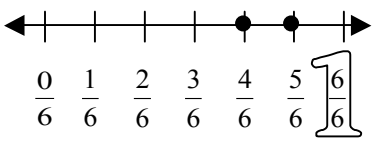
*3rd grade denominators are 2, 3, 4, 6, and 8. Fourth grade denominators are 2, 3, 4, 5, 6, 8, 10, 12, 100. You can include 4th grade denominators in your you tries as an extension.

Sample You Try: $\frac{4}{6}$ and $\frac{5}{6}$

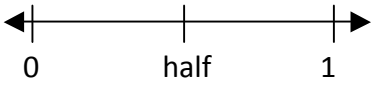
$\frac{4}{6} < \frac{5}{6}$

Student Notes

	1 whole					
$\frac{4}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



Compare Fractions to 1 Half on a Number Line.
This sets students up for using 1 half as a benchmark.

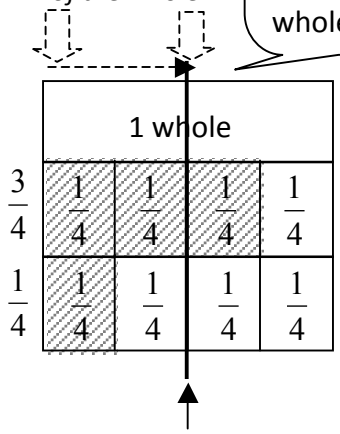


We Do: **Compare $\frac{3}{4}$ and $\frac{1}{4}$ to 1 half on a number line.**

Run pencil along the top of the whole.

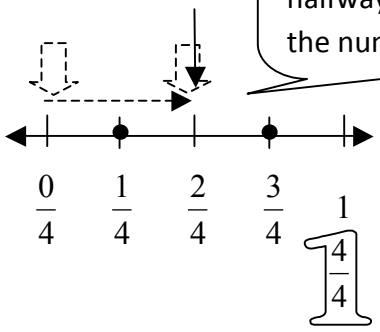
Raise your hand when I get halfway across the whole.

Use your model drawing and number line. Decide as a class if 1 fourth is greater or less than 1 half. Same for 3 fourths. Having them find the halfway mark can help.

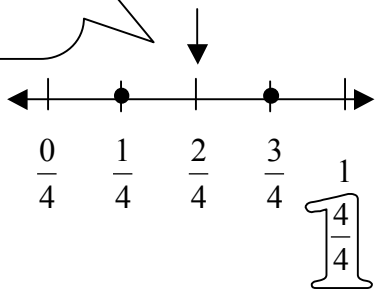


Is 1 fourth greater than 1 half? Share with your neighbor how you know. Is 3 fourths…..

Raise your hand when I get halfway across the number line.

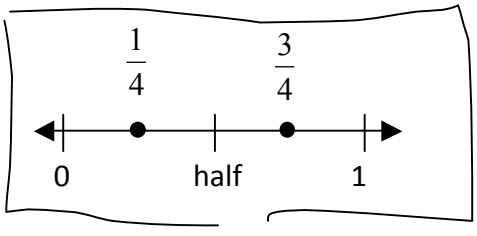


So…1 fourth is on the left of 1 half and 3 fourths is on the right.



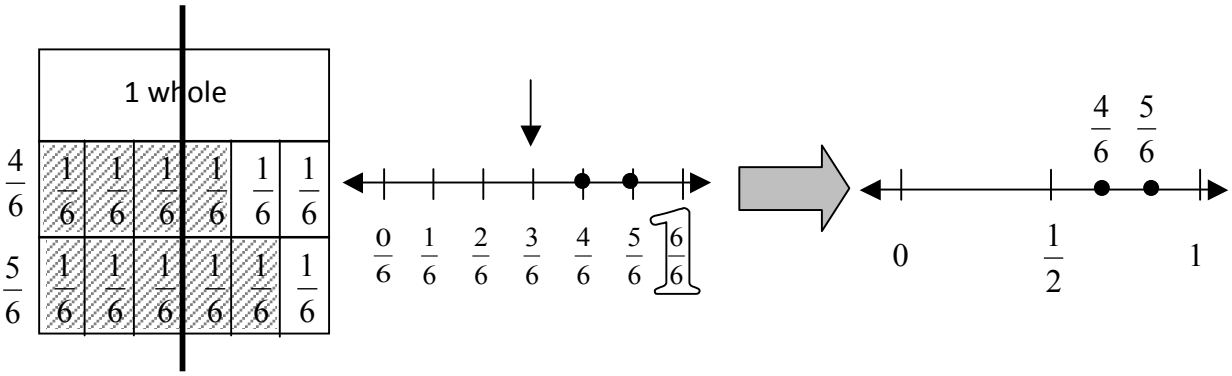
Establish new number line and mark each fraction accordingly. Students can copy the new number line in their notebooks.

Student Notes:



You Try: $\frac{4}{6}$ and $\frac{5}{6}$ (from previous lesson)

Use student example from the previous day. Students draw their own number lines (on a new page or on their work from the day before) and plot 4 sixths and 5 sixths. The important thing is that the fractions are plotted on the correct side of 1 half and that they are on the correct side of each other (example, 4 sixths needs to be to the left of 5 sixths, and both need to be on the right of 1 half.)



Additional You Tries: Show students pre-drawn models and number lines. Have them show on a number line whether each fraction is less than or more than half.

Part 2 Overview: Compare Unit Fractions

Connection to Base Ten Concepts

1 ten < 1 hundred

1 thousand > 1 one

1 hundred < 1 thousand

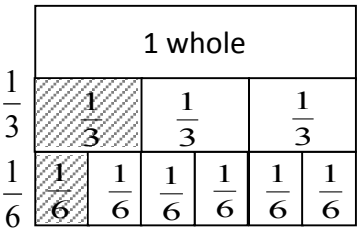
Use What You Know: Compare Unit Fractions

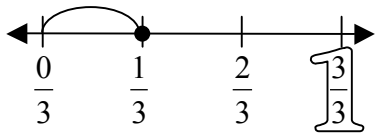
$\frac{1}{3} > \frac{1}{6}$

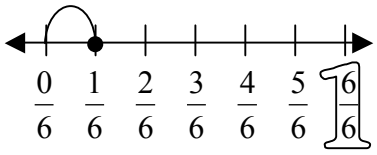
1 third > 1 sixth

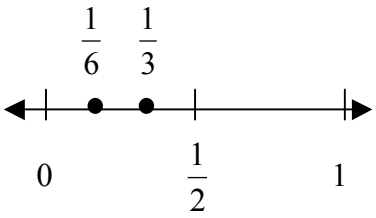
I can tell from the denominators that 1 third is a larger unit than 1 sixth, but I'm not sure how much larger. I think 1 third is greater than 1 sixth.

Bar Model and Number Lines

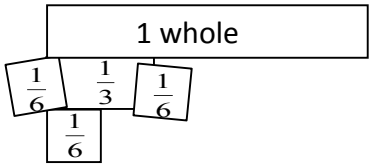




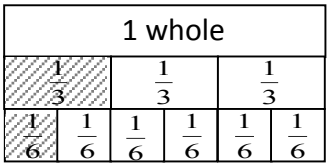




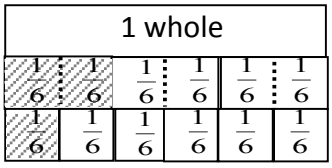
Next Steps Grade 4: Convert to Like Denominators



We know 1 third is greater than 1 sixth because thirds are larger units than sixths. We also know that they are both less than 1 half. Let's look at it another way.

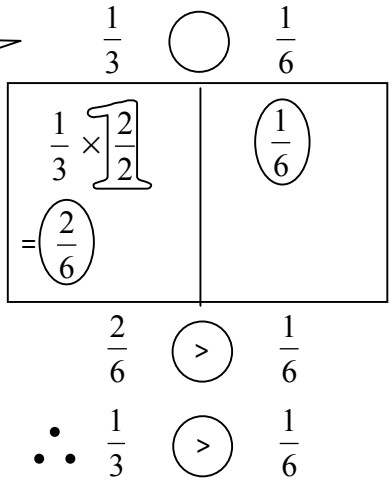


Start with different size units.



Convert to same size units.

Compare same size units.



Part 2: Compare Unit Fractions

So far, we've focused on comparing same size units.

Now we are moving into comparing units with different sizes.

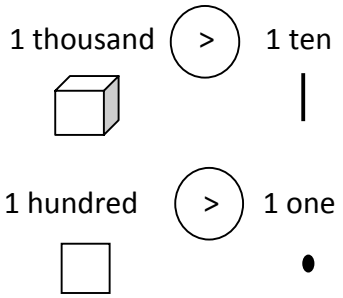
Let's go back again to what you already know about comparing GREATER numbers.

You already know that 1 thousand is greater than 1 ten.

You already know that 1 hundred is greater than 1 "one".

What's the same about each side? Share with your neighbor what you think. (only one unit on each side)

What's different about each side? Share with your neighbor what you think. (one side has a bigger unit)



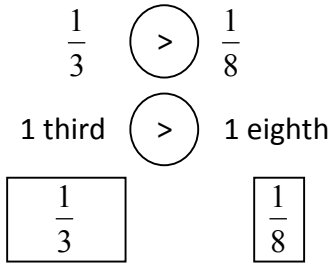
How does this relate to fractions?

Sometimes we compare unit fractions, like 1 third and 1 eighth. If we know which piece, or unit, is bigger, then it's easier for us to figure it out.

Let's compare 1 third and 1 eighth. Thirds and eighths are different sizes, but which is bigger?

Raise your hand if already know which is bigger. How do you know? (Some kids will know from experience, but may not know how to explain it.)

Let's write what (student name) thinks. Now we'll check it with the models… Done!

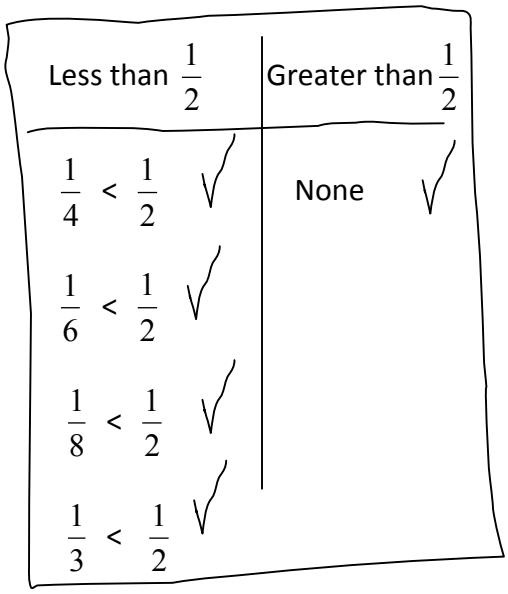


So…if we can learn which units are bigger and which are smaller, then that will make it easier to compare different fractions.

Using Fraction Tiles

Have students work in partners to compare unit fractions using fraction tiles. Isolate one unit fraction, for example 1 half, and compare the other unit fractions to it. Then switch to a new target unit fraction. Some students with more confidence can predict first, then prove their answers with tiles. Others can discover the answer with the tiles. In either case, they record their work.

Possible Student Notes:



*3rd grade denominators are 2, 3, 4, 6, and 8. Fourth grade denominators are 2, 3, 4, 5, 6, 8, 10, 12, and 100. You can include 4th grade denominators in your you tries as an extension.

Use the data from the student comparisons of 1 half to discover how to decide if a unit fraction is larger or smaller than another unit fraction.

One Way…

Put student work on the document reader:

What do you notice about 1 half? What is it greater than? What is it less than? Share with your neighbor. Walk around and notice student observations.

Call on a student(Which fractions are less than 1 half? (1 fourth, 1 sixth, 1 eighth, and 1 third)

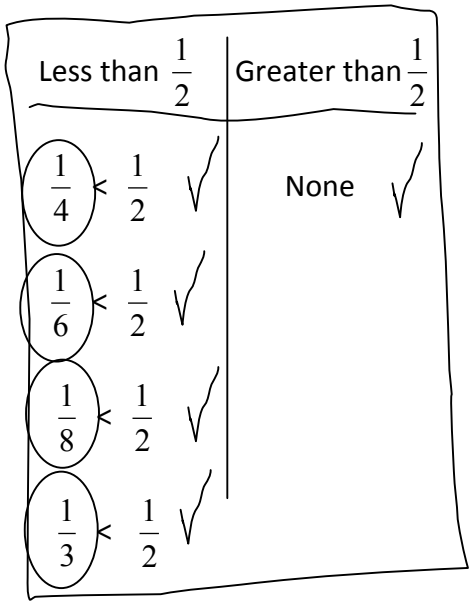
What is 1 half less than? (none).Really?!

Why is that? Think. Share with a neighbor.

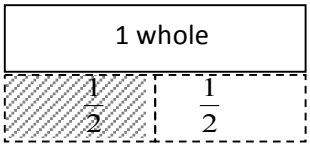
If a student is willing, he/she can share out. Otherwise…

Let's find out.

Possible Student Notes:

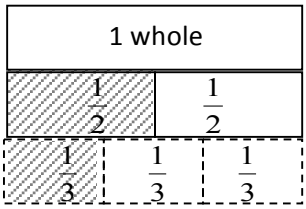


*(Draw on the board…)*We start with 1 whole. You said the biggest unit was 1 half.



How many halves in a whole? (2) Okay 2 pieces.

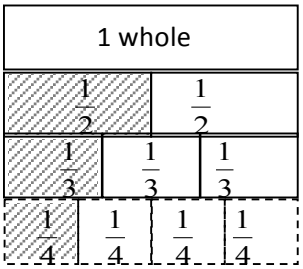
What was next largest? Check with your partner. Thumbs up when you have it. Everyone. (thirds) How many thirds (3) Okay. 3 pieces.



What's next?...(.fourths),

Then what? ….(.sixths),

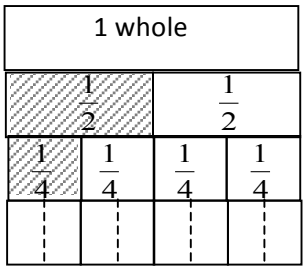
Then what? (eighths)



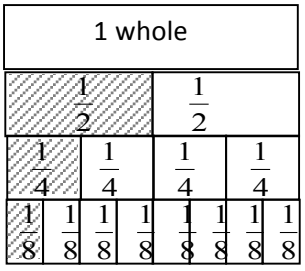
I don't want you to have to watch me draw the whole time, so I'm just going to add in the fourths.

Think….why are fourths less than halves? Share with your neighbor. Quiet hand… (because there are more pieces in the whole) Thumbs up if you agree. What if we broke the whole into tenths, or ten pieces? Would the pieces be bigger or smaller? (smaller) What about hundredths, or a hundred pieces? (smaller)

I think this is important stuff, so let's put this in our notebooks. Let's start with 1 whole, like I did. Then divide into 2 pieces. For our notebooks, we're going to skip to fourths because that's easier to do at first. Now let's go to the smallest unit…the eighths.



If we extend the fourths down to the next layer, we will have 4 sections. But I need 8. What should I do? If I divide each fourth into 2 pieces, then will I have 8? Let's check…2, 4, 6, 8. Yes, it works.



So…we said that …If there are a lot of pieces in the whole, the size of one piece is small.

If there are only a few pieces in the whole, the size of one piece is big.

Do you agree?(Yes, of course!)

What part of the fraction tells us how many pieces in the whole? $\frac{1}{4}$ ← Is it here? (no)

← Is it here? (yes)What do we call that part? (the denominator)

That's good stuff. Let's write that down in our notes:

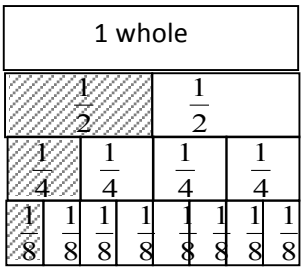
$\frac{1}{4}$

The denominator tells us the number of pieces in the whole.

A greater denominator means a smaller piece.

A smaller denominator means a larger piece.

Possible Student Notes



$\frac{1}{4}$

The denominator tells us the number of pieces in the whole.

A greater denominator means more pieces which means smaller pieces.

A smaller denominator means fewer pieces which means larger pieces.

Models Drawings and Number Lines

We Do: Compare $\frac{1}{3}$ and $\frac{1}{6}$

Think for a second. Which piece do you think is greater? Share with your neighbor. Thumbs up for 1 third? Thumbs up for 1 sixth. Okay.

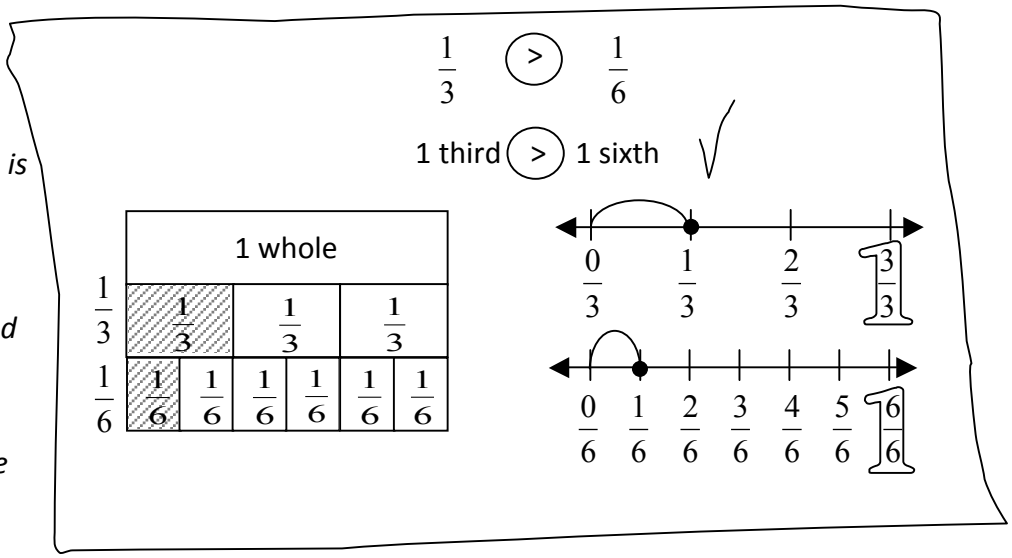
Let's look at the denominator. *Note the denominator for each. Note the number of pieces in the whole.*

Let's make a prediction. *It doesn't matter if you predict correctly or incorrectly, as long as you go back and model checking your answer.*

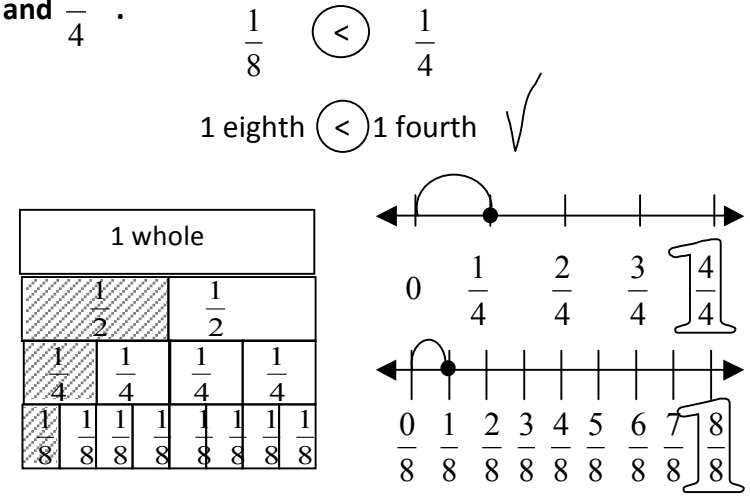
Possible Student Notes:

Walk kids through model drawing an number lines as needed. Notice which is greater.

Remind students that both number lines need to be the same length because they are working with the same whole.



Sample You Try: Compare $\frac{1}{8}$ and $\frac{1}{4}$.

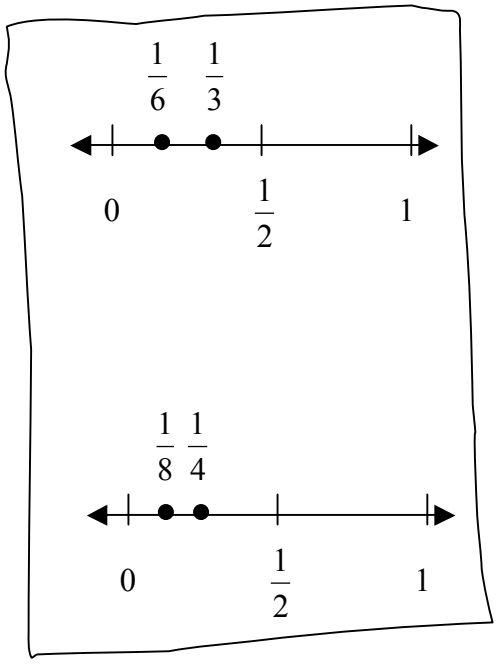


After a few more you tries, add the layer of comparing to 1 half. Go back to your notes for the We Do: (1 third and 1 sixth)

Student Notes

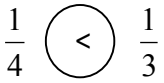
Walk kids through comparing to 1 half. Some ways to help them: Use your picture or number lines to help. Also, remember that before we said that all the unit fractions (1 sixth, 1 eighth, etc. were less than 1 half.) Students write in their notes.

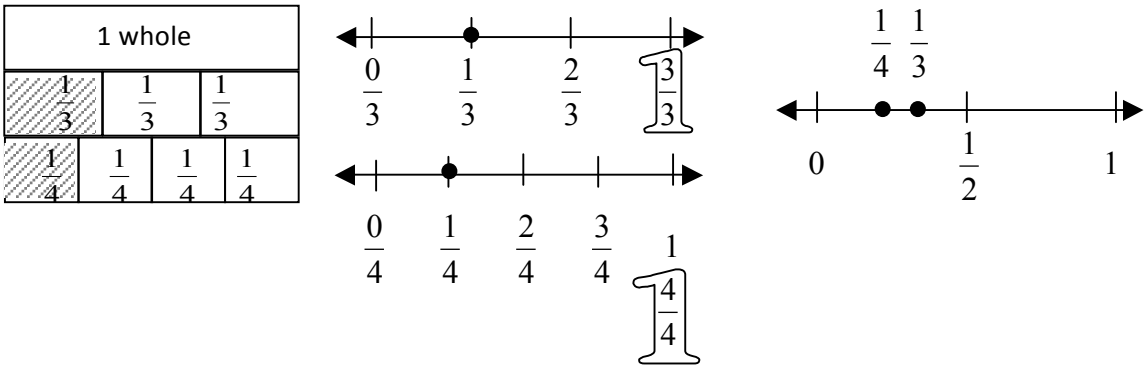
Pull out the first you try (1 eighth and 1 fourth). Review with the students as needed. Have them try plotting 1 eighth and 1 fourth on the new number line themselves.



After that, students can incorporate model drawings, and both sets of number lines in their you tries.

Some fractions are more difficult for students to draw. For example, 1 third and 1 fourth. It is good to give students guidance and experience trying to draw these models. There are also pre-drawn models at the end of the lesson that can be used as needed.



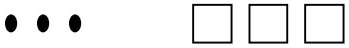


Part 3 Overview: Unlike Denominators, Like Numerators

Connection to Base Ten Concepts

5 tens > 5 ones

3 ones < 3 hundreds

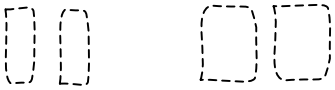


2 tens < 2 thousands

Use What You Know: Compare Same Number of Different Size Units

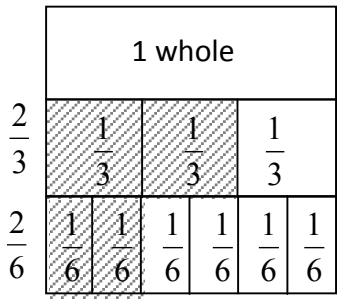
$\frac{2}{6} < \frac{2}{3}$

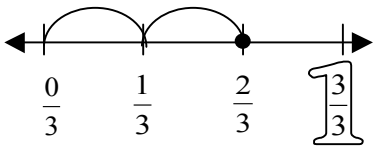
2 sixths < 2 thirds

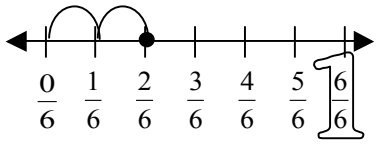


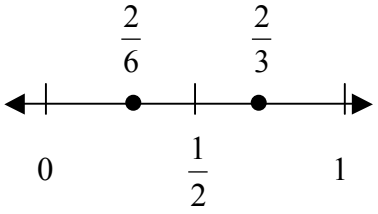
Sixths are smaller units than thirds. There are 2 smaller units and 2 larger units. The 2 smaller units are less than the 2 larger units.

Bar Model and Number Lines

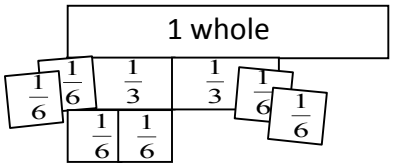




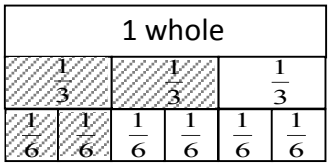




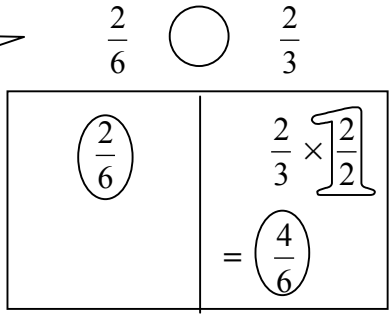
Next Steps Grade 4: Convert to Like Denominators

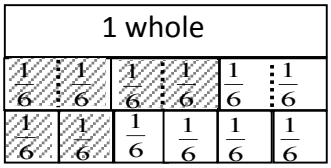


We know 2 sixths is less than 2 thirds because sixths are smaller units than thirds. Also, 2 sixths is less than 1 half and 2 thirds is greater than 1 half. Let's look at it another way.



Start with different size units.





Convert to same size units.

Compare same size units.

$\frac{2}{6} < \frac{4}{6}$

$\therefore \frac{2}{6} < \frac{2}{3}$

Part 3: Unlike Denominators, Like Numerators

Now we can compare same size units and different size units.

Now we're moving into more complex fractions.

Let's go back again to what you already know about comparing GREATER numbers.

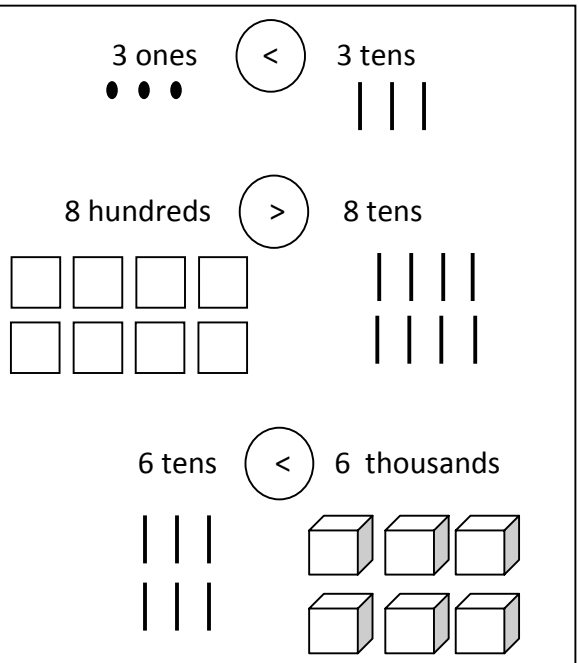
You know that 3 ones is less than 3 tens.

You know that 8 hundreds is greater than 8 tens.

You know that 6 tens is less than 6 thousands.

What's the same about each side? (same number of units on each side)

What's different about each side? (one side has a bigger unit)

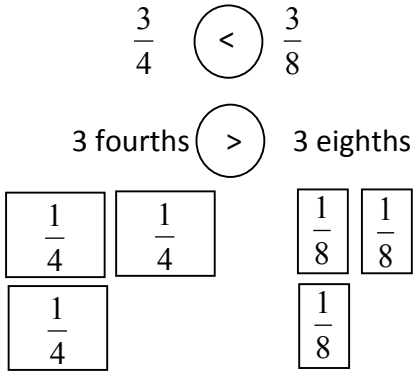


How does this relate to fractions?

Now that we are more familiar with the size of different unit fractions like 1 fourth and 1 eighth, we can compare other fractions like 3 fourths and 3 eighths.

Which is bigger, fourths or eighths? (fourths) How do you know? (eighths have more pieces in the whole, so the pieces are smaller) *Show them 1 fourth and 1 eighth.* You are correct!

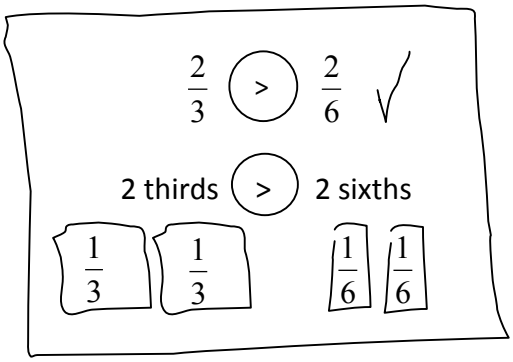
So…if I have 3 big pieces (3 fourths) on one side and 3 small pieces (3 eighths) on the other, then which is greater? (3 fourths) *Take out 2 more fourths and eighths to confirm they are correct.*



Using Models (As Necessary)

We Do: **Compare $\frac{2}{3}$ and $\frac{2}{6}$.**

Student Notes



Where do we look to find out how big the pieces are? Numerator? (no) Denominator? (yes). What is the denominator here? (3) Which makes thirds. What is the denominator here? (6) which makes what?(sixths). Which are bigger, thirds or sixths? (thirds) *Show models to verify.*

So which is greater 2 thirds or 2 sixths? (2 thirds). *A drawing might help:*

Sometimes drawing a sketch can help. Draw 2 big pieces and label them thirds. Now draw 2 small pieces for sixths. They don't have to be perfect, but try to make the big pieces about the same size and the small pieces about the same size. Can you tell from my picture which amount is greater? (Yes of course!) (2 thirds) Now test with models and write a check in your notes to show you proved your answer.

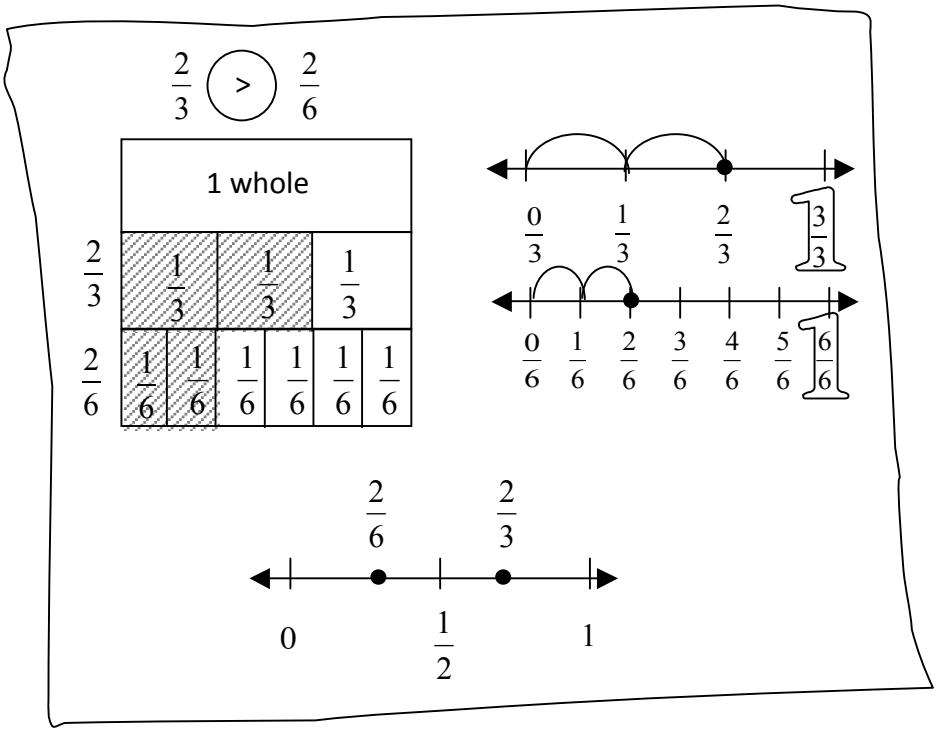
You Tries: *Have students compare several pairs of fractions with common numerators greater than 1 as they did in the We Do. The sketch might work for some students, but it's not necessary.*

Models Drawings and Number Lines

Student Notes

You Try: Students draw a model and both types of number lines to compare 2 thirds and 2 sixths.

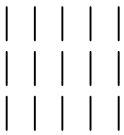
Other You Tries: Have students do other examples of fractions with unlike denominators and like numerators.



Grade 4: Unlike Denominators, Unlike Numerators

Connection to Base Ten Concepts

15 tens $<$ 2 hundreds



2 tens $<$ 25 ones

When we compare tens to hundreds, we can decompose the hundreds into tens, or build the tens into hundreds. That way we can compare the same size units.

Start with What You Know:

Compare Different Amounts of Different Size Units

$\frac{2}{5}$? $\frac{6}{10}$

2 fifths ? 6 tenths

I know fifths are larger pieces than tenths.

I know 2 fifths are greater than 2 tenths, but there are 6 tenths. It could go either way.

I can relate the fractions to 1 half or convert to common denominators or common numerators.

Convert to Like Denominators

$\frac{2}{5} \bigcirc \frac{6}{10}$

$\frac{2}{5} \times \frac{2}{2} = \frac{4}{10}$ $\quad \frac{6}{10}$

$\frac{4}{10} < \frac{6}{10}$

$\therefore \frac{2}{5} < \frac{6}{10}$

Convert to Like Numerators

$\frac{2}{5} \bigcirc \frac{6}{10}$

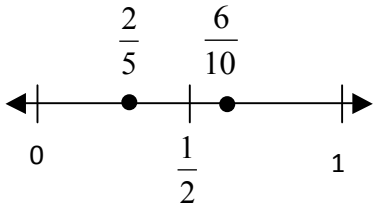
$\frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$ $\quad \frac{6}{10}$

$\frac{6}{15} < \frac{6}{10}$

$\therefore \frac{2}{5} < \frac{6}{10}$

I know that fifteenths are smaller than tenths. There are the same number of each unit. 6 fifteenths is less than 6 tenths, and 2 fifths is less than 6 tenths.

Compare to One Half



2 fifths is less than 1 half, and 6 tenths is greater than 1 half.

$\therefore \frac{2}{5} < \frac{6}{10}$

Assessment:

Which if the following is true about $\frac{5}{6}$ and $\frac{5}{8}$?

Circle each answer that is correct. Fill in and label the correct model drawing.

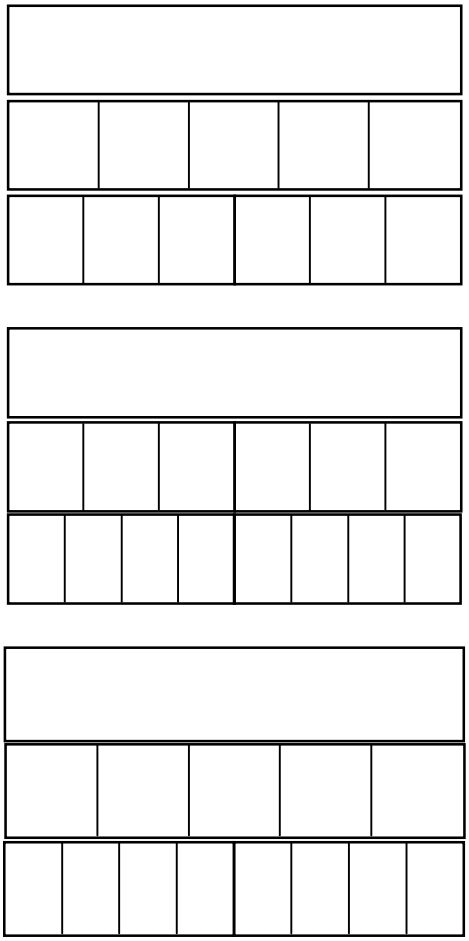
A. $\frac{5}{6}$ is greater than $\frac{5}{8}$

B. $\frac{5}{6}$ has fewer pieces than $\frac{5}{8}$

C. The pieces in $\frac{5}{6}$ are smaller.

D. $\frac{5}{6}$ is more than half of the whole.

E. $\frac{5}{8}$ is more than half of the whole.

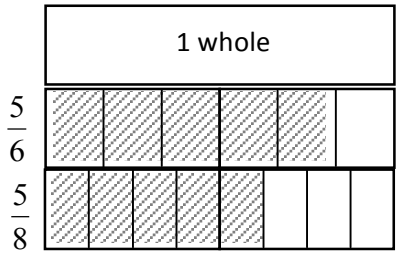


Answer Key: Correct Answers: A, D, E.

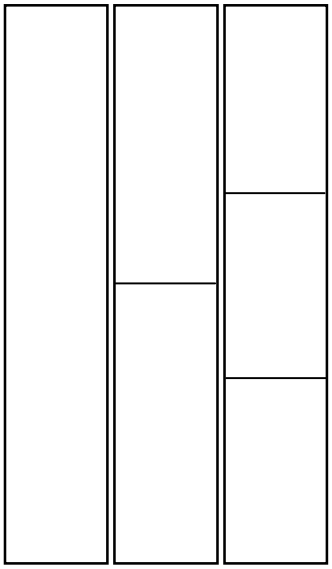
B: Student my look to the denominator to count the number of pieces.

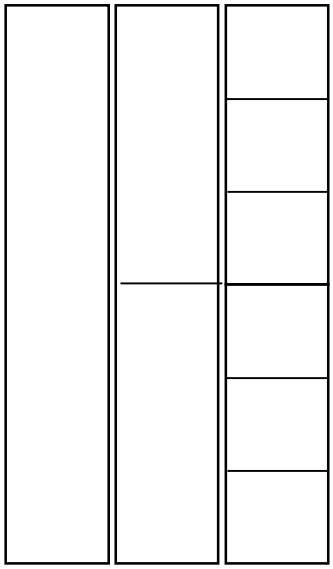
C: Student may see the denominator 6 as an indicator of smaller pieces.

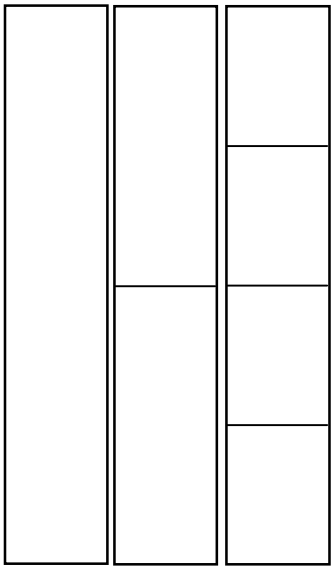
Center Drawing Correct:

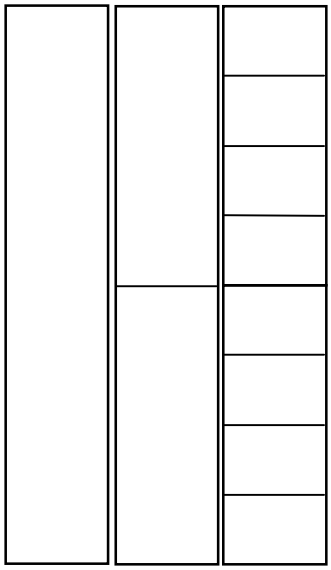


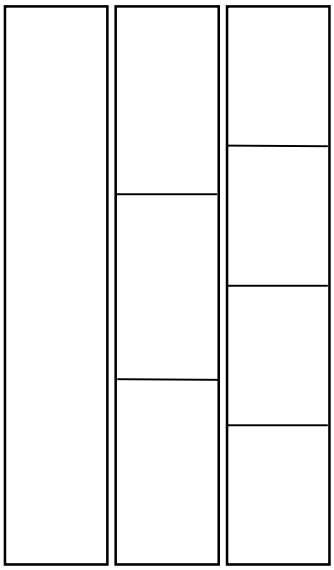
If the student chose the other drawings, he/she does not understand the role of the numerator and denominator on a basic level.

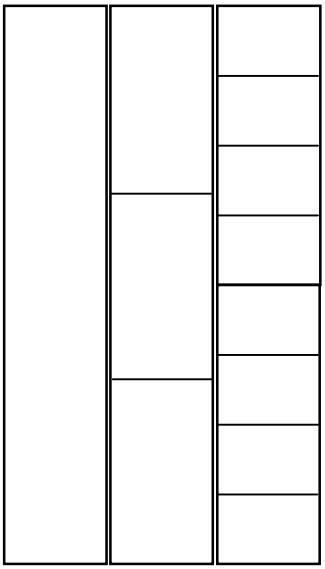


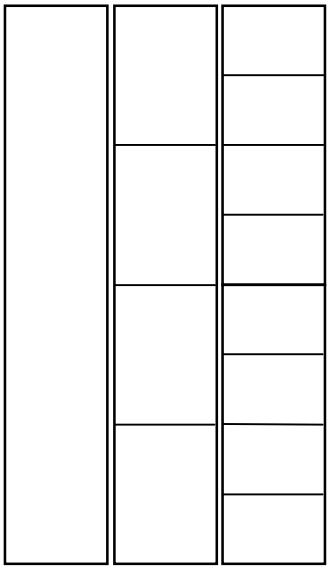


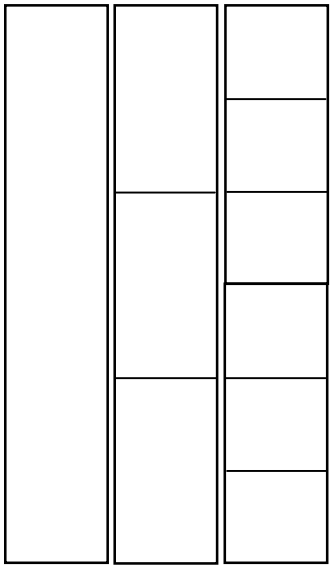


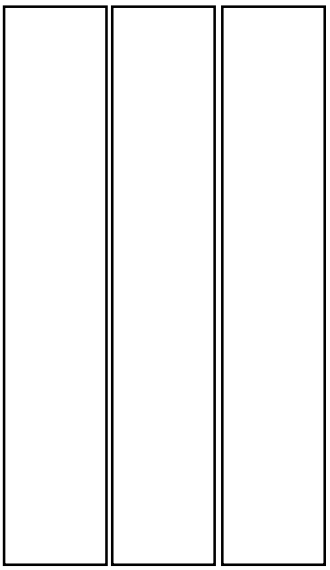


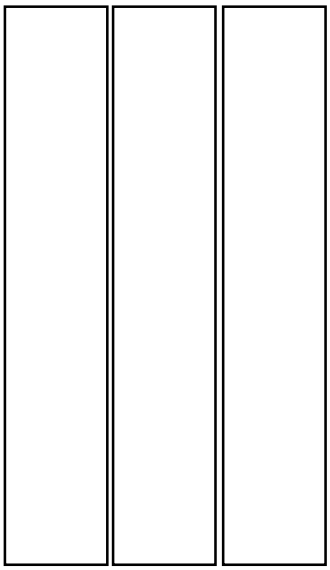


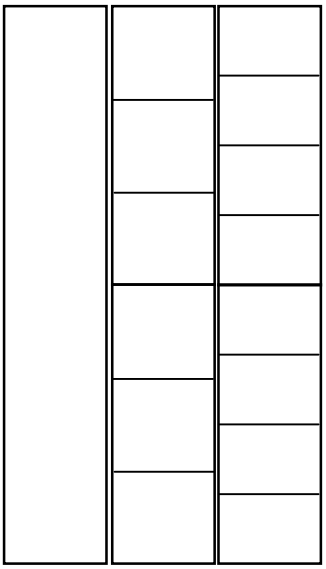


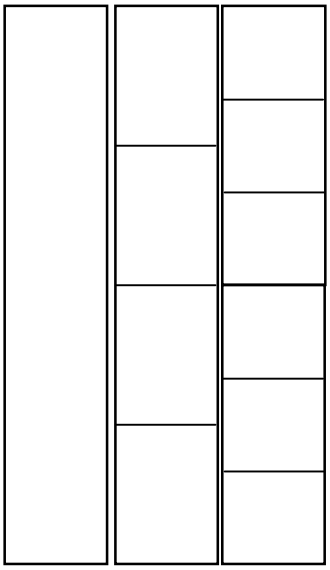












Warm Up

Selected Response:

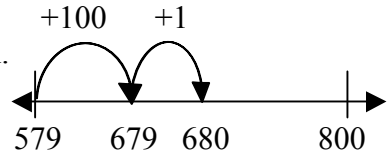
Mr. Hixon's class was working on this problem: 800 – 579.

Then the bell rang, and they had to go to recess.

Which of the following **unfinished** problems are done correctly **so far?**

800 - 579

A.



Yes ○ No ○

B.

$$\begin{array}{r} 800 \\ -579 \end{array} \rightarrow \begin{array}{r} 899+1 \\ -579 \end{array}$$

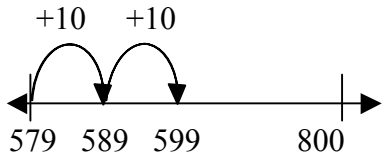
Yes ○ No ○

C.

$$\begin{array}{r} 800 \\ -579 \end{array} \rightarrow \begin{array}{r} 799 \\ -579 \end{array}$$

Yes ○ No ○

D.



Yes ○ No ○

Current:

Gina finished 5 sixths of her homework. Draw a model of 5 sixths. Then plot 5 sixths on a number line.

Review:

For the following problem:

*Draw a bar model.

*Find the quotient.

*Check with multiplication.

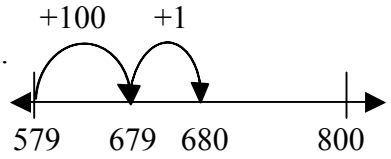
$$\frac{87}{3} = \square$$

Warm Up Debrief:

Have students debrief the warm up as much as possible. For the selected response, they can justify why they chose yes or no. For the others, they can show their work and explain what they did. Having them show their work encourages neat work.

Selected Response:

A.



Yes ● No ○

B. $800 \rightarrow 899+1$

$-579 \rightarrow -579$

Yes ○ No ●

800 is incorrectly decomposed into 899 + 1, which is greater than 800.

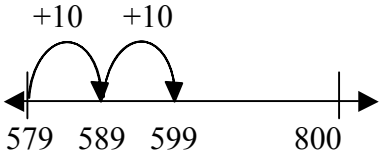
C. $800 \rightarrow 799$

$-579 \rightarrow -579$

Yes ○ No ●

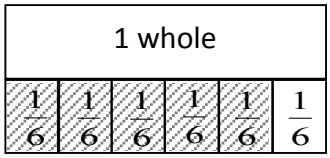
800 is incorrectly decomposed into 799, which is less than 800.

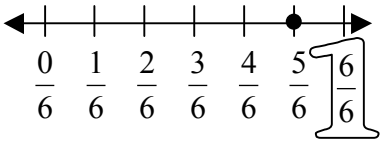
D.



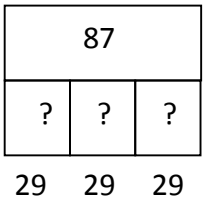
Yes ● No ○

Current:





Review:



$3\overline{)87}$

-60 | 20

27

-27 | 9

0

20 + 9 = 29

Check:

$$\begin{array}{r} 29 \\ \times\ 3 \\ \hline 27 \\ +\ 60 \\ \hline 87 \end{array} \quad \begin{array}{l} \\ \\ = 3 \times 9 \\ = 3 \times 20 \\ \\ \end{array}$$

$\therefore \quad \frac{87}{3} = 29$